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EXCITATION OF LOW-FREQUENCY FIELDS IN A MULTIMEMBRANE CHAMBER*

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The problem of the excitation of a given field of ideal fluid velocities and accelerations is considered, when the fluid fills a chamber which is small compared with the wavelength. An oscillatory flow is excited by flexible membranes in the chamber walls. The membrane oscillations are realized by the periodic injection and drainage of fluid into and from compartments behind the membranes. Low-frequency excitation of a liquid phase medium in a space whose linear dimensions are less than the excitation wavelength is used for a variety of technological processes /1/. It is then important to ensure, not only given energy characteristics of the oscillatory flow, but also a pre-assigned distribution of the field of fluid velocities and accelerations.

1. A rectangular chamber $D_0 = \{x, y, z: 0 < x < L_1, 0 < y < L_2, 0 < z < L_3\}$ is filled with an ideal fluid of density ρ_1. On the chamber walls there are hatches for loading and unloading, which are interpreted as free fluid surfaces, and where membranes are mounted. Into the spaces D_n $(n = 1, \ldots, 2N)$ behind the membranes, ideal fluid of density ρ_2 is periodically injected and removed, with period $2\pi/\omega$, where ω is the angular frequency. The variable pressure of the periodic fluid flow in domains D_n excites oscillations of the membranes. These oscillations are transformed into periodic oscillatory flow of the fluid in domain D_0. It is assumed that $\omega L c^{-1} \ll 1$, where L is the characteristic dimension of the chamber, c is the velocity of sound in a fluid of density ρ_1, and $vc_1^{-1} \ll 1$, where v and c_1 are the fluid velocity modulus and the velocity of sound in the fluid in the compartments D_n behind the membranes.

The potential φ of the velocity field in domain D_0 is the solution of the following problem:

$$\Delta\varphi = 0; \ \partial\varphi/\partial n = 0 \text{ on } \Gamma \tag{1.1}$$

Here $\partial/\partial n$ is the derivative with respect to the outward normal, Γ is the part of the boundary of D_0 formed by the rigid walls, $\partial\varphi/\partial z = -i\omega Z_j$, $Z_j = i\omega g^{-1}\varphi$ on the free surfaces $\Gamma_{1j} = \{x, y, z: 0 < x < L_1, \ l_{1j} < y < l_{2j}, \ j = 0, 1, \ l_{10} = 0, \ l_{21} = L_2, \ z = L_3\}$, where Z_j is the deviation of the free surface from the equilibrium position, $\partial\omega/\partial n = -i\omega w_n$, $p_n = -q_n$ on the mean surface of the n-th membrane, q_n is the load on the mean surface of a membrane, w_n is the normal component of the sag of the n-th membrane, and p is the pressure on the mean surface from the fluid of density ρ_1.

In the compartments D_n the flow velocity potentials φ_n satisfy Poisson's equation

$$\Delta\varphi_n = \sum_{m=1}^{M_n} Q_{nm} e^{i\theta_n} \delta(x - \xi_{0m}^n, \ y - \eta_{0m}^n, z - \zeta_{0m}^n) e^{-i\omega t} \tag{1.2}$$
$$n = 1, \ldots, 2N$$

where $(\xi_{0m}^n, \ \eta_{0m}^n, \ \zeta_{0m}^n)$ are the coordinates of the sources with deliveries Q_{nm} and delay phases θ_n, $\delta(x)$ is the delta function, and t is time, which is a parameter of the problem. On the rigid walls we have the no-flow condition, and on the membranes, the matching condition $\partial\varphi_n/\partial n = -i\omega w_n$, $p_n = -q_n^1$, where p_n and q_n^1 are the pressure and load on the mean membrane surface from the second fluid.

The pressure in domains D_0 and D_n $(n = 1, \ldots, 2N)$, is found from the linearized equations of motion

$$\begin{aligned} p - p_0 - \rho_1 g (L_3 - z) &= i\omega\rho_1 \varphi(x, y, z) e^{-i\omega t} \\ p_n - p_{n0} - \rho_2 g (L_n - z) &= i\omega\rho_2\varphi_n(x, y, z) e^{-i\omega t}, \ L_n = \max_{(x, y, z) \in D_n} z \end{aligned} \tag{1.3}$$

(p_0, p_{n0} are the pressures in the working chamber and the n-th compartment at zero sag of the membrane and g is the vertical component of the acceleration due to gravity).

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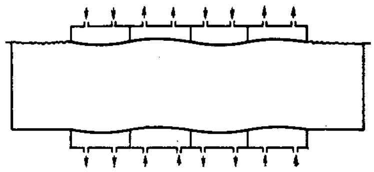

Fig.1

We consider the problem of the excitation in a subdomain of D_0 of an oscillatory flow with given vertical velocity and acceleration, with modulus not less than a pre-assigned amount. For the excitation we locate membranes in the lower and upper walls of the working chamber, strictly one below the other, the free surfaces Γ_{1j} being located above the rigid part of the lower wall (Fig.1).

We know that, at small non-zero displacements of the membranes, only the vertical component of the sag w_n, which, in the domain Γ_n, formed by the undeformed mean surface of the n-th membrane and the bounded piecewise smooth closed curve γ_n, satisfies the equations /2/

$$(D\nabla^4 - \omega^2\rho\delta)w_n = p_n - p, \quad n = 1, \ldots, N \tag{1.4}$$
$$(D\nabla^4 - \omega^2\rho\delta)w_n = p - p_n, \quad n = N+1, \ldots, 2N$$
$$\nabla^4 = (\partial^2/\partial x^2 + \partial^2/\partial y^2)^2, \quad D = E\delta^3/[12(1-\nu^2)]$$

where N is the number of membranes on the lower (upper) wall of the working chamber, and δ, ρ, E, ν are the membrane thickness, the density, the modulus of elasticity, and Poisson's ratio of the membrane material respectively. The time factor $e^{-i\omega t}$ is omitted.

The boundary condition on the curves γ_n can be written as

$$w_n = 0, \quad \mu_n = 0 \text{ on } \gamma_n \tag{1.5}$$

where μ_n is the bending moment in the direction of the normal.

We shall further assume that domains Γ_n $(n = 2, \ldots, N)$, are, with $z = 0$, translations of the domain Γ_1 by n steps of length l along the y axis, while domain Γ_n $(n = N+2, \ldots, 2N)$, with $z = L_3$, are translations of the domain Γ_{N+1} by n steps of length l along the y axis.

2. Let $\mathbf{V}^*(x, y, z)e^{-i\omega t}$, $\mathbf{a}^*(x, y, z)e^{-i\omega t}$ be the pre-assigned velocity and acceleration fields of the oscillatory fluid flow inside the chamber, which are realized by the sags w_n^* of the membranes and satisfy the conditions

$$|V_z(x, y, z)| \geqslant V^*, \quad |a_z^*(x, y, z)| \geqslant A^*; (x, y, z) \in D^* \subset D_0 \tag{2.1}$$

Here, D^* is the uniton of cylinders $D^* = U_n D_n^*$ with bases Γ_n^*, which are subdomains of Γ_n of the upper and lower membranes, and V^* and A^* are constants.

We pose the following problem: it is required to choose the coordinates of the centres of the sources, the number of them, and the volume deliveries Q_{nm}, in such a way that the distribution field of the velocity V_z in domain D_0 satisfies the condition

$$\max_{x, y, z} |V_z(x, y, z) - V_z^*(x, y, z)| \leqslant \varepsilon, \quad (x, y, z) \in D^* \tag{2.2}$$

This problem is an example of an inverse problem of oscillatory flow of fluid in a bounded volume. The general theory of such problems is treated in /3/.

Let $\varphi^*(x, y, z)$ be the potential of the velocity $V^*(x, y, z)$, and G the Neumann function for the domain D_0. Using Green's formula and the matching conditions on the membranes Γ_n, we obtain the following expression for the potential φ^* in terms of the sags w_n^*

$$\varphi^* = -i\omega I^* + i\omega J^* \tag{2.3}$$
$$I^* = \sum_{n=1}^{N} \int_{\Gamma_n} w_n^* G\,ds - \sum_{n=N+1}^{2N} \int_{\Gamma_n} w_n^* G\,ds, \quad J^* = \sum_{j=0}^{1} \int_{\Gamma_{1j}} Z_j^* G\,ds$$

Since the conditions $-i\omega w_n^* = V_{zn}^*$ hold on membranes $\Gamma_n, n = 1, \ldots, 2N$, the sags w_n^* will be assumed to be known, and the potential φ^* is given by

$$\varphi^* = \sum_{n=1}^{N} \int_{\Gamma_n} V_{zn}^* G\,ds - \sum_{n=N+1}^{2N} \int_{\Gamma_n} V_{zn}^* G\,ds + i\omega J^*$$

The deviation Z_j^* of the free surface can then be assumed to be given, or Z_j^* can be found from the system of integral equations

$$Z_j^* + \omega^2 g^{-1} J^* = \omega^2 g^{-1} I^*, \quad j = 0, 1 \tag{2.4}$$

If $\{R_{ij}\}_{i,j}$ is the solving kernel of system (2.4), we can write the deviation Z_j^* of the free surface as

$$Z_j^* = -\omega^4 g^{-2} \sum_{i=0}^{1} \int_{\Gamma_{1i}} R_{ij} I^* \, ds + \omega^2 g^{-1} I^* \tag{2.5}$$

Let G_n be the Neumann function for the domain D_n, the flow velocity potential φ_n in the domain D_n will be calculated from the expression

$$\varphi_n(x, y, z) = \sum_{m=1}^{M_n} \exp(i\theta_n) Q_{nm} G_n(x, y, z, \xi_{0m}^n, \eta_{0m}^n, \zeta_{0m}^n) \pm \tag{2.6}$$
$$i\omega I_n, \quad I_n = \int_{\Gamma_n} w_n G_n d_s$$

where the upper sign is taken for $n = 1, \ldots, N$, and the lower one for $n = N+1, \ldots, 2N$.

We find the potential φ, excited by the sags w_n, from an expression similar to (2.3), in which the w_n^* are replaced by w_n. On substituting into system (1.4) the values of the pressure, found from (2.3) and (2.6), we obtain the system of integrodifferential relations connecting the membrane sags and the deviations Z_j of the free surfaces with deliveries Q_{nm}:

$$(D\nabla^4 - \omega^2\rho\delta) w_k \mp \omega^2\rho_1 [J - I] + \omega^2\rho_2 I_k = \tag{2.7}$$
$$\pm i\omega\rho_2 \exp(i\theta_k) \sum_{m=1}^{M_k} Q_{km} G_k(\xi_k, \eta_k, \zeta_k, \xi_{0m}^k, \eta_{0m}^k, \zeta_{0m}^k)$$
$$Z_j + \omega^2 g^{-1} (J - I) = 0, \ j = 0, 1$$
$$\zeta_k = 0, \ k = 1, \ldots, N; \ \zeta_k = L_3, \ k = N+1, \ldots, 2N$$
$$J = J^*, \ I = I^* \text{ for } w = w^*, \ z = z^*$$

(the upper sign is taken for $k = 1, \ldots, N$, and the lower one for $k = N+1, \ldots, 2N$). If the deliveries are known, (2.7) is a system of integrodifferential and integral equations for finding the sags w_k and the deviations Z_j. In our present problem, however, the deliveries Q_{km} are unknown, and have to be found.

We require that the unknown sags w_k should be equal to the sags w_k^*; then it follows from the last equations of system (2.7) for $j = 0, 1$, and from system (2.4), that $Z_j^* = Z_j$, while the flow potential φ is equal to the pre-assigned potential φ^* /4/. We substitute the values w_n^* instead of w_n into system (2.7). We can then interpret (2.7) as an approximation of the known function on the left-hand side of (2.7) by a sequence of known functions

$$G_{km}(\xi_k, \eta_k) = G_k(\xi_k, \eta_k, \zeta_k, \xi_{0m}^k, \eta_{0m}^k, \zeta_{0m}^k)$$
$$m = 1, \ldots, M_k$$

with unknown coefficients Q_{km}. Since the system of functions G_{km} is linearly independent on Γ_k /5/, the problem is solvable and the coefficients of the best approximation of Q_{km} are given by

$$\sum_{m=1}^{M_k} Q_{km} \int_{\Gamma_k} G_{km} G_{kq} \, ds = b_{kq}, \quad q = 1, \ldots, M_k \tag{2.8}$$
$$b_{kq} = \mp \omega^{-1}\rho_2^{-1} \exp(-i\theta_k) \int_{\Gamma_k} G_{kq} [(D\nabla^4 - \omega^2\rho\delta) w_k^* \mp$$
$$\omega^2\rho_1 (J^* - I^*) + \omega^2\rho_2 I^*_k] \, ds$$

(the upper sign is taken for $k = 1, \ldots, N$, and the lower one for $k = N+1, \ldots, 2N$).

Let B denote the operator given by the integrodifferential expression on the left-hand side of system (2.7) with $k = 1, \ldots, 2N$, in which the Z_j are given by (2.5), which is specified in the set of sufficiently smooth functions which satisfy boundary conditions (1.5). The number M_k of sources is found from

$$\left\| Bw^* - \sum_{m=1}^{M_k} [\pm i\omega\rho_2 \exp(i\theta_k) Q_{km} G_{km}] \right\| \leqslant \| B \| \, \| w^* - w_* \|$$

where w_* is the sag realized by the given distribution of sources with deliveries Q_{km}. Assume that the parameter ω is not a natural frequency of oscillation of the membrane-fluid system. We then have the estimate $\| w^* - w_* \| \leqslant \varepsilon_1 \| B \|^{-1} = \varepsilon$. In order to satisfy condition (2.1), we need to know the distribution of the vertical component of the velocity field of our solution with respect to the z coordinate.

3. As an example, consider the excitation of a given fluid velocity and acceleration

field in the domain $D_0 = \{0 < x < L,\ 0 < y < 6L,\ 0 < z < L/2\}$ for an eight-membrane chamber with membranes measuring $L \times L$ with thickness δ, located one above the other on the lower and upper walls, and loaded and unloaded by compartments measuring $L \times L$, located on the upper wall.

To find the class of functions $\mathbf{V}^*(x, y, z)$ and $\mathbf{a}^*(x, y, z)$, for which the problem of the excitation of a given field satisfying conditions (2.1) has a solution, we define the structure of the field in terms of the membrane sags w_n. In other words, we first solve the direct problem, when the parameter $M_n = 1$ in Eq.(1.2), i.e., there is one source with delivery $Q_n = Q$ and $\xi_{0m}^n = L/2$, $\eta_{0m}^n = L/2 + nL$, $\zeta_{0m}^n = -h$, $n = 1, \ldots, 4$, $\zeta_{0m}^n = L/2 + h$, $n = 5, \ldots, 8$. The boundary conditions (1.5) can be written for this case as

$$
\begin{aligned}
&w_n = \partial^2 w_n/\partial x^2 = 0 \text{ for } x = 0,\ L\\
&w_n = \partial^2 w_n/\partial y^2 = 0 \text{ for } y = nL,\ (n+1)L \text{ for } n = 1, \ldots, 4\\
&y = (n-4)L,\ (n-3)L \text{ for } n = 5, \ldots, 8
\end{aligned} \tag{3.1}
$$

In system (2.7) we make the change of variables $x_1 = x/L$, $y_1 = y/L$, $z_1 = z/L$, and we introduce the notation $w_{n1} = w_n/\delta$, $Z_{j1} = Z_j/\delta$, $G^1 = LG$, $G_n^{\,1} = LG_n$, $K^4 = L^4\omega^2\rho\delta/D$. For simplicity, the index unity will henceforth be omitted.

We shall seek the solution of system (2.7) of integrodifferential equations by the Bubnov-Galerkin method. In view of boundary conditions (3.1), the sags of the lower and upper membranes may be written as

$$
w_n = \sum_{q,\, m=1}^{N_n} A_{qm}^n \sin \pi q\xi \sin \pi m\eta_n \tag{3.2}
$$

$$
(\eta_n = \eta - n \text{ for } n = 1, \ldots, 4,\quad \eta_n = \eta - (n-4) \text{ for } n = 5, \ldots, 8)
$$

and the deviation of the free surface may be written as /6/

$$
Z_j = \sum_{q,\, m=1}^{N} C_{qm}^j \cos \pi q\xi \cos \pi m\eta_j,\quad \eta_j = \eta - 5j,\quad j = 0, 1 \tag{3.3}
$$

We substitute (3.2) and (3.3) into system (2.7). From the orthogonality conditions we obtain a system of algebraic equations for the coefficients A_{qm}^n and C_{qm}^n, which has a solution at least in the case of low-frequency excitation.

Consider the case when $\omega < \omega_1$. Let the main contribution to the velocity field distribution be from the first harmonic of the sag w_n. In this case we can find the amplitude $A_{11}^{\ n}$ from the condition for the interior Neumann problem to be solvable for domain D_n:

$$
\begin{aligned}
&A_{11}^{\ n} = A_{11}^{n+4},\ A_{11}^{\ n} = i\pi^2 \exp(i\theta_n)\, Q\, [4\omega L^2\delta]^{-1},\ n = 1, \ldots, 4\\
&C_{11}^{\ j} = 0,\ j = 0, 1
\end{aligned}
$$

The first natural frequency of fluid oscillation in domain D_0 is given by the Bubnov-Galerkin method by the relation

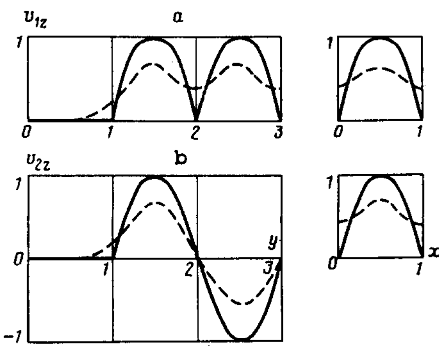

Fig.2

$$
\begin{aligned}
\omega_1 = {} & \frac{2\pi^2}{L^2}\left(\frac{D}{\rho\delta}\right)^{1/2} \Bigg\{1 - \frac{16^2 \rho_1 L}{\pi^5 \rho\delta} \sum_{n,\, m=0}^{\infty} (2n+1)^2 (-1)^m \cos^2 \times\\
& \frac{\pi(2m+1)}{12} \cos\frac{\pi(2m+1)}{4} [(2n+1)^2 - 4]^{-2} [1 - (2m + 1/6)^2]^{-1} \times\\
& \sin^2 \frac{\pi(2m+1)}{6} [(2n+1)^2 + (2m + 1/6)^2]^{-1/2} \operatorname{th}\left\{\frac{\pi}{4}[(2n+1)^2 + (2m + 1/6)^2]^{1/2}\right\}\Bigg\}^{-1/2}
\end{aligned}
$$

The velocity field potential is found from (2.2), (2.3), in which we put $m,\ q = 1$:

$$
\begin{aligned}
\varphi(x, y, z) = {} & -\frac{16 i\omega L\delta}{\pi^3} \sum_{q=1}^{4} A_{11}^{\ q} \sideset{}{^*}\sum_{n,\, m=0}^{\infty} \cos 2\pi nx \cos\frac{\pi m y}{6} \times\\
& \cos\frac{\pi m(2q+1)}{12} \left\{(1+\delta_{0n})(1+\delta_{0m})(4n^2-1)\left[1 - \left(\frac{m}{6}\right)^2\right]\right\}^{-1} \times\\
& \cos\frac{\pi m}{12} \operatorname{sh}\left[\frac{\pi(1-4z)}{2}\left\{n^2 + \left(\frac{m}{12}\right)^2\right\}^{1/2}\right] \times
\end{aligned}
$$

$$[(12n)^2 + m^2[^{-1/2}\mathrm{ch}\left[\frac{\pi}{2}\left\{n^2 + \left(\frac{m}{12}\right)^2\right\}^{1/2}\right]$$
$$\delta_{00} = 1,\ \delta_{0n} = 0$$

The symbol Σ^* means that the term of the sum with $m = 6$ is zero.

The rate of oscillatory flow is given for the first case of excitation with $\theta_k = 0$ and for the second case with $\theta_k = \pi(k+1)$ by the relations

$$\frac{V_{1z}}{i\omega\delta A_{11}} = v_{1z} = \frac{32}{3\pi^2}\sum_{n,\,m=0}^{\infty}{}^{*}(-1)^m \cos 2\pi nx \cos\frac{\pi my}{3}\times$$
$$\cos^2\frac{\pi m}{6}\left\{(1+\delta_{0n})(1+\delta_{0m})(4n^2-1)\left[1-\left(\frac{m}{3}\right)^2\right]\right\}\times$$
$$\cos\frac{\pi m}{3}\mathrm{ch}\left\{\frac{\pi(1-4z)}{2}\left[n^2+\left(\frac{m}{6}\right)^2\right]^{1/2}\right\}\mathrm{ch}^{-1}\left[\frac{\pi}{2}\left\{n^2+\left(\frac{m}{6}\right)^2\right\}^{1/2}\right]$$

the term of the sum with $m = 3$ being zero

$$\frac{V_{2z}}{i\omega\delta A_{11}} = v_{2z} = \frac{8}{3\pi^2}\sum_{n,\,m=0}^{\infty}{}^{*}(-1)^m \cos 2\pi nx \cos\pi(2m+1)\frac{y}{6}\times$$
$$\sin\frac{\pi(2m+1)}{3}\left[(1+\delta_{0n})(4n^2-1)\left\{1-\left[\frac{2m+1}{6}\right]^2\right\}\right]^{-1}\times$$
$$\mathrm{ch}\left\{\frac{\pi(1-4z)}{2}\left[n^2+\left(\frac{2m+1}{12}\right)^2\right]^{1/2}\right\}\mathrm{ch}^{-1}\left\{\frac{\pi}{2}\left[n^2+\left(\frac{2m+1}{12}\right)^2\right]^{1/2}\right\}$$

The results of calculating the distribution of the vertical velocity v_z in domain D_0 are shown in Fig.2,a for the first case of excitation, and in Fig.2,b for the second case (by the symmetry, only for the set of values $\{0 \leqslant x \leqslant 1/2,\ 0 \leqslant y \leqslant 3,\ 0 \leqslant z \leqslant 1/4\}$). The continuous line is the distribution of the vertical velocity component distribution for $z = 0$, and the broken line, for $z = 1/4$.

The distribution field is characterized by the following properties. The velocity $V_z(\xi_k, \eta_k, z=0)$, $(\xi_k, \eta_k) \in \Gamma_k$ is the same, apart from a constant, as the sag. As z increases, the maximum value $|V_z(1/2, k+1/2, 0)|$ decreases to $|V_z(1/2, k+1/2, 1/4)|$, while on the membrane boundary it increases from zero to the value $|V_z(\xi_k, \eta_k, 1/4)|$, where $\xi_k, \eta_k \in \gamma_k$. For every point $(\xi_k, \eta_k) \in \Gamma_k$ the minimum with respect to z of velocity V_z is equal to

$$V_z^* = \min\,[|V_z(\xi_k, \eta_k, 0)|,\ |V_z(\xi_k, \eta_k, 1/4)|]$$

For analytic studies the following approximation is useful:

$$V_z^* = \alpha V_z(x, y, 0),\ \alpha = \min_k |V_z(1/2, k+1/2, 1/4)/(\omega A)|$$

For convex sags w_n, symmetric about the membrane centre, the distribution field of the velocity vertical projection behaves in the same way as in the above case of one-mode sag. Since V_z^* is known as a function of the sag, the problem of constructing a given velocity field that satisfies conditions (2.1), reduces to the problem of constructing a given membrane sag w_k^* such that, in the domain Γ_k,

$$|w_k^*| \geqslant V^*/(\omega\delta\alpha) \tag{3.4}$$

To be specific, let us consider the problem of forming the sag w_k of the lower membranes. Put

$$Q_{km}^* = i\omega\rho_2\exp(i\theta_k)L^3D^{-1}\delta^{-1}Q_{km}$$
$$f_k^* = (\nabla^4 - K^4)w_k^* - \rho_1 LK^4\rho^{-1}\delta^{-1}[J^* - I^*] + \rho_2 LK^4\rho^{-1}\delta^{-1}I_k^*,$$
$$I_k^* = I_k \text{ for } w_k^* = w_k$$

We substitute into the k-th equation of system (2.7), $k = 1, \ldots, 4$, the value

$$w_k^* = \sum_{n,\,q}^{N_k} A_{nq}^k \sin\pi n\xi_k \sin\pi q\eta_k$$

which satisfies condition (3.4) with $(\xi_k, \eta_k) \in \bar{\Gamma}_k$, and the deviation Z_j^* of the free surface, as given by (2.5). As a result, we have

$$\sum_{m=1}^{M_k} Q_{km}^* G_k(\xi_k, \eta_k, 0, \xi_{0m}^k, \eta_{0m}^k, -hL^{-1}) = f_k^*,\quad k = 1, \ldots, N \tag{3.5}$$

For the approximation of the function f_k^* we can use (2.8).

Consider the special case when an extra condition is imposed on the amplitude of the sag

w_k^*:

$$\sum_{n,\,q=1}^{N_k}[(-1)^n-1][(-1)^q-1]\,A_{nq}[\pi^4(n^2+q^2)^2- \tag{3.6}$$
$$K^4(1+\rho_2L\rho^{-1}\delta^{-1})]\,q^{-1}h^{-1}=0$$

The approximation problem (3.5) can then best be solved by comparing the Fourier coefficients of the left-hand and right-hand sides with respect to the system of functions

$$\cos\pi r\xi_k\cos\pi n\eta_k;\ 0\leqslant r,\ n\leqslant R-1,\ r^2+n^2\neq 0,\ M_k=R^2$$

We obtain

$$\sum_{m=1}^{M_k}Q_{km}^{*}\cos\pi r\xi_{0m}^k\cos\pi n\eta_{0m}^k=d_{rn} \tag{3.7}$$

$$d_{rn}=-\pi(r^2+n^2)^{1/2}\operatorname{sh}[\pi h(r^2+n^2)^{1/2}L^{-1}]\times$$
$$\int_{\Gamma_k}f_k^*\cos\pi r\xi_k\cos\pi n\eta_k\,ds$$

We add to system (3.7) the equation obtained from the solvability condition for the interior Neumann problem in domain D_k and conditions (3.6)

$$\sum_{m=1}^{M_k}Q_{km}^k=d_{00},\quad d_{00}=\sum_{n,\,q=1}^{N_k}[(-1)^n-1]\times \tag{3.8}$$
$$[(-1)^q-1]\,A_{nq}[\pi^4(n^2+q^2)^2-K^4]\,q^{-1}n^{-1}\pi^{-2}$$

We choose the source coordinates $\xi_{0m}^k,\ \eta_{0m}^k$ in such a way that we have

$$\xi_{01}^{\ k}=\xi_{0(R+1)}^k=\ldots=\xi_{0[R(R-1)+1]}^k=1/(R+1),\ldots$$
$$\xi_{0R}^k=\xi_{02R}^k=\ldots=\xi_{0RR}^k=R/(R+1)$$
$$\eta_{01}^{\ k}=\ldots=\eta_{0R}^k=1/(R+1),\quad \eta_{0[(R(R-1)+1]}^k=\ldots$$
$$=\eta_{0RR}^k=R/(R+1)$$

i.e., the source centres form a square mesh on the lower wall of domain D_k. In this case the determinant of system (3.7), (3.8) is the Kronecker product of Van der Monde determinants. We write the solution by using Cramer's rule

$$Q_{km}^k=\frac{\Delta_m^{\ k}}{\Delta^k},\quad \Delta^k=\Big[4^{(R-1)^2}\prod_{1\leqslant i<j\leqslant R}\sin\frac{i+j}{2(R+1)}\sin\frac{i-j}{2(R+1)}\Big]^{2R}$$

($\Delta_m^{\ k}$ is the determinant obtained by replacing the m-th column of Δ^k by the right-hand side $d_{rn}=(d_{00},d_{10},\ldots,d_{R-1,0},\ldots,d_{R-1,R-1})^{\mathrm T}$). The number of sources $M_k=R^2$ is found from the condition

$$\Big|\sum_{r,\,n=R}^{\infty}\int_{\Gamma_k}f_k^*\cos\pi r\xi_k\cos\pi n\eta_k\,ds\Big|\leqslant\varepsilon_1$$

In the linear statement, the acceleration of the oscillatory flow is given by $\mathbf a=-i\omega\mathbf V$, so that the construction of a given acceleration field is similar to the construction of a given velocity field.

4. Consider the case of resonant excitation of the flow, assuming that the fluid in the working chamber and in the compartments behind the membranes is ideal, and that energy is dissipated only in the membrane material. While remaining within the theory of small sags, we introduce into the integrodifferential equations of membrane motion and the motion of the free surfaces, in accordance with /7/, the supplementary term

$$D\nabla^4w_n^*+\rho\delta\frac{\partial^2w_n^*}{\partial t^2}=\pm\rho_2\varepsilon\sum_{m=1}^{M_n}Q_{nm}G_n(\xi,\eta,\zeta_n,\xi_{0m}^n,\eta_{0m}^n,\zeta_n^*)\,\omega\sin\omega t\pm \tag{4.1}$$
$$\rho_1gZ_i^*+\rho_2\int_{\Gamma_n}G_n\frac{\partial^2w_n^*}{\partial t^2}\,ds+\varepsilon D\Phi(w_n^*),\quad n=1,\ldots,2N$$

$$Z_i^* = -g^{-1}\Big[\sum_{q=1}^{N}\int_{\Gamma_q} G\frac{\partial^2 w_q^*}{\partial t^2}\,ds - \sum_{q=N+1}^{2N}\int_{\Gamma_q} G\frac{\partial^2 w_q^*}{\partial t^2}\,ds - \sum_{j=1}^{2}\int_{\Gamma_{1j}}\frac{G\partial^2 Z_j^*}{\partial t^2}\,ds\Big], \quad i=1,2$$

In (4.1), the upper sign is taken for $n = 1, \ldots, N$, and the lower one for $n = N+1 \ldots, 2N$, while the functional $\Phi(w_n^*)$ characterizes the hysteresis-type energy dissipation in the membrane material. The function Φ is determined experimentally. It is assumed that the disturbing force is small, i.e., the deliveries Q_{km} must have the order $Q_{km} \sim \varepsilon A_{km}$, $A_{km} = O(1)$ as $\varepsilon \to 0$. The energy loss in the membrane material leads to a phase shift of the membrane oscillations, and hence to a phase shift of the oscillations of the fluid filling the working chamber.

We seek the solution of system (4.1) as

$$w_n^* = uw_n(x, y)\cos\tau + \varepsilon u_1(x, y, \tau) + \ldots, \quad Z_j^* = uZ_j(x, y)\cos\tau$$
$$\omega^2 = \omega_1^2 + \varepsilon\Delta_1 + \ldots, \quad \psi = \psi_0 + \varepsilon\psi_1 + \ldots, \quad \tau = \omega t + \psi \tag{4.2}$$

For clarity, we take the resonant frequency ω_1 calculated in Sect.3. We require that the functions $u_1, \ldots,$ do not contain the principal harmonics $\cos\tau$ and $\sin\tau$. Substituting expansion (4.2) into system (4.1) and comparing like powers of ε, we obtain the system of equations for ω_1, Δ_1, and ψ_0:

$$(D\nabla^4 - \omega_1^2\rho\delta)\,w_n = \pm\rho_2\omega_1^2(I - J) + \rho_2\omega_1^2 I_n, \quad n = 1, \ldots, 2N \tag{4.3}$$
$$Z_j = \omega^2 g^{-1}[I - J], \quad j = 1, 2$$

$$D\nabla^4 u_1 + \rho\delta\left[-u\Delta_1 w_n\cos\tau + \omega_1^2\partial^2 u_1/\partial\tau^2\right] = \tag{4.4}$$
$$\pm\rho_2\sum_{m=1}^{M_n} Q_{nm}G_n(\xi, \eta, \zeta_n, \xi_{0m}^n, \eta_{0m}^n, \zeta_{0m}^n)\sin(\tau - \psi_0) +$$
$$\rho_2\int_{\Gamma_n}\left[-uw_n\Delta_1\cos\tau + \omega_1^2\partial^2 u_1/\partial\tau^2\right]G_n\,ds + \varepsilon D\Phi(u, w_n, \tau)$$
$$\varepsilon\Phi(w_n^*) = \varepsilon\Phi(u, w_n, \tau) + \varepsilon^2, \quad n = 1, \ldots, N$$

System (4.3) is the same as the homogeneous system (2.7), so that ω_1 can be regarded as the first natural frequency of system (2.7).

We use the method of harmonic balance to find the unknowns Δ_1 and ψ_0. For this, we multiply Eqs.(4.4) by $w_n\cos\tau$ and $w_n\sin\tau$, and integrate over the membrane surface during the complete time cycle. After transformations, we obtain the system

$$\cos\psi_0 = \frac{1}{S}\int_{\Gamma_n}\int_0^{2\pi}\varepsilon D\Phi(u, w_n, \tau)\,w_n\sin\tau\,d\tau\,ds \tag{4.5}$$
$$\Delta = \Big[S\sin\psi_0 - \int_{\Gamma_n}\int_0^{2\pi}\varepsilon D\Phi(u, w_n, \tau)\,w_n\cos\tau\,d\tau\,ds\Big]\Big[\pi\rho\delta\int_{\Gamma_n}uw_n^2\,ds -$$
$$\pi D\int_{\Gamma_n}w_n\,ds\int_{\Gamma_n}uw_nG_n(\xi, \eta, \zeta_n, \xi_1, \eta_1, \zeta_{1n})\,ds\Big]^{-1}$$
$$S = \pi\rho_2\sum_{m=1}^{M_n}Q_{nm}\omega_1\int_{\Gamma_n}G_n(\xi, \eta, \zeta_n, \xi_{0m}^n, \eta_{0m}^n, \zeta_n^\circ)\,w_n\,ds$$

In (4.5) we take as w_n the eigenfunction corresponding to the eigenvalue ω_1. Solving system (4.5) simultaneously and using the known expression for $\Phi(u, w_n, \tau)$, we can plot the resonance curve $u = f(w)$. To refine the sag, we find the function u_1 from (4.4). After finding the w_n, the velocity field is calculated from (2.3).

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AN ANALYTICAL SOLUTION OF THE PROBLEM OF CONVECTIVE DIFFUSION IN THE NEIGHBOURHOOD OF A DISCONTINUITY OF THE CATALYTIC PROPERTIES OF A SURFACE*

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The problem of convective diffusion when a binary mixture flows round a plate when there is a line of discontinuity of the catalytic properties on the plate is considered. The effect of longitudinal diffusion is taken into account. The surface is assumed to be non-catalytic up to the discontinuity but possesses a finite catalytic activity after the discontinuity. At low values of the coefficient of catalytic activity, an analytic solution of the problem is obtained by the application of a Fourier transform. The asymptotic forms of the solution are found in the form of simple formulae both near the remote from the point of discontinuity of the boundary conditions and both upstream and downstream. A comparison is made with the solutions obtained in the boundary layer approximation and by a numerical method /1/.

The problem of convective diffusion (or thermal conductivity) in the case of a transition from a not-catalytic surface onto an ideally catalytic surface has been solved /2, 3/ by the Wiener-Hopf method.

1. The stationary flow of a two component incompressible liquid or gas with constant diffusion properties and a linear velocity profile $(u' = VL^{-1}y',\ v' = 0)$ in the x' direction around an infinite plate $y' = 0$, on the surface of which a heterogeneous first-order reaction occurs, is considered. The surface is assumed to be non-catalytic in the half-plane $y' = 0, x' < 0$ and to possess a finite catalytic activity in the half plane $y' = 0, x' > 0$.

The diffusion equation (which is identical in form to the heat conduction equation) and the boundary conditions in this case have the form

$$y\frac{\partial c}{\partial x} = \frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2}, \quad -\infty < x < \infty, \quad y > 0 \tag{1.1}$$

$$x \to -\infty,\ \forall y \ \text{ and } y \to \infty,\ \forall x{:}\ c \to c_0 \tag{1.2}$$

$$y = 0, \quad x < 0{:}\ \frac{\partial c}{\partial y} = 0; \quad y = 0, \quad x > 0{:}\ \frac{\partial c}{\partial y} = kc;$$

$$k = \left(\frac{L}{VD}\right)^{1/3} k'$$

Here k' is the rate constant for the heterogeneous recombination on the surface, and the dimensionless variables x and y are related to the dimensional variables in the following manner:

$$x' = \left(\frac{DL}{V}\right)^{1/3} x, \quad y' = \left(\frac{DL}{V}\right)^{1/3} y$$

(D is the coefficient of diffusion and c is the concentration).

Let us consider the case when $k \ll 1$. A solution of the problem can then be sought in the form

$$c = c_0 (1 - kf + \ldots) \tag{1.3}$$

For the function $f(x, y)$ we obtain an equation, which is identical to (1.1), while the boundary conditions take the form

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